Introduction to Data Communications (COMP 3721)

Instructor: Maryam Tanha

Fall 2021

Learning Outcomes of This Lecture

- **By the end of this lecture you will be able to**
  - Explain the Pulse Code Modulation (PCM) technique for analog-to-digital conversion.

Agenda

- Introduction
- Analog-To-Digital Conversion
- Summary

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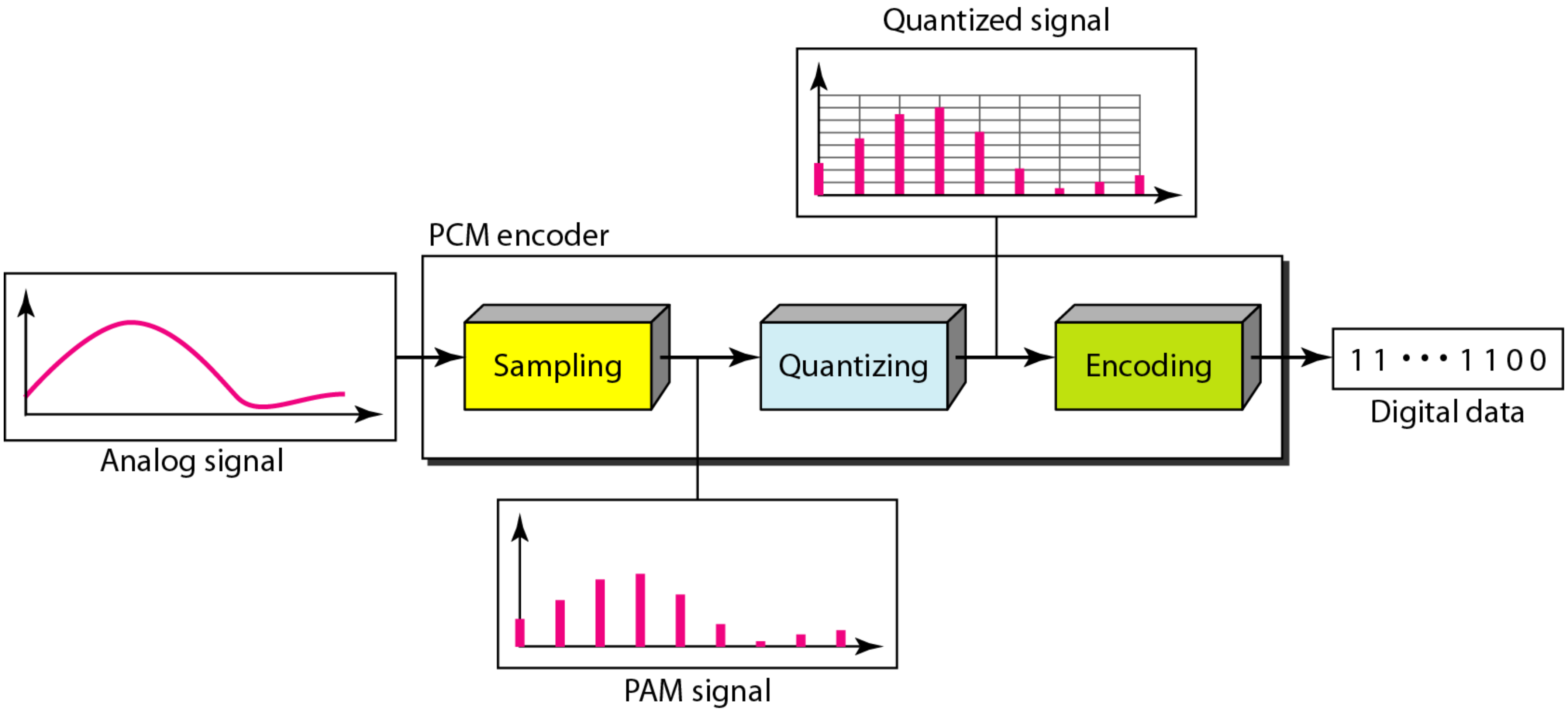
Analog-To-Digital Conversion

- **Digitization**
  - converting an analog signal to digital data

Analog-To-Digital Conversion

- **Digitization**
  - converting an analog signal to digital data
- **Pulse Code Modulation (PCM)**
  - the most commonly used technique for digitization.

PCM


Quantized signal
PCM encoder
Sampling
Quantizing
Encoding
1 1 • • • 1 1 0 0
Digital data
Analog signal
PAM signal

Sampling

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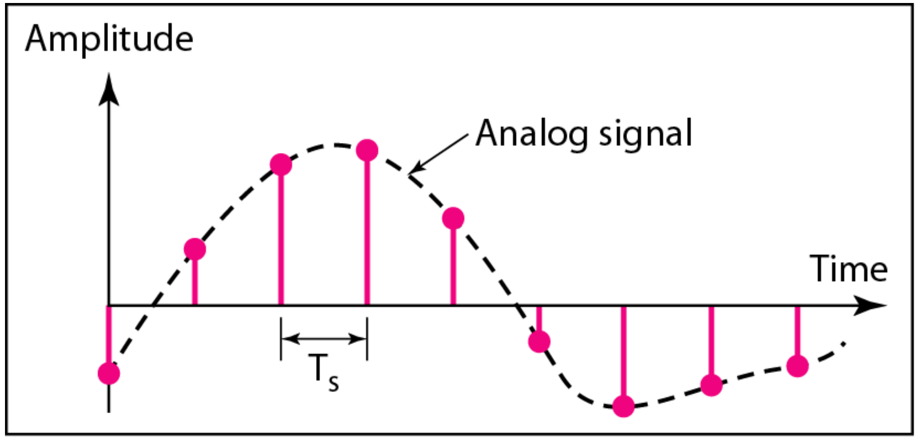
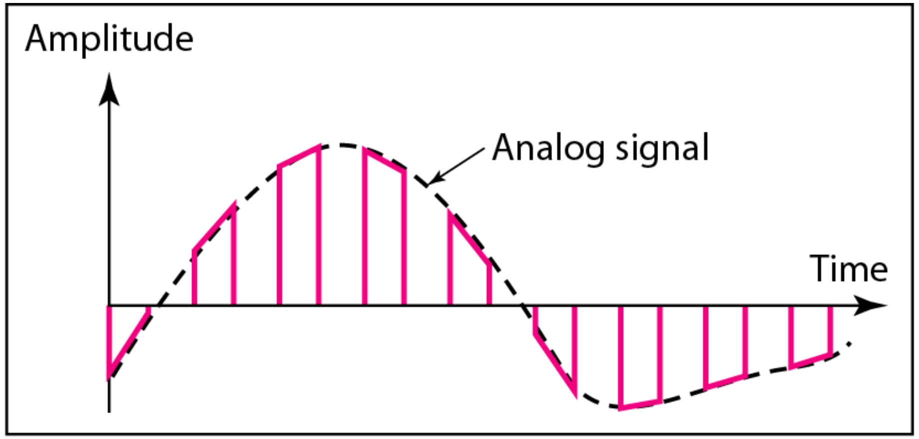
The result of sampling is still an analog signal with non-integral values (a series of pulses, with amplitude values between the maximum and minimum amplitudes of the signal)

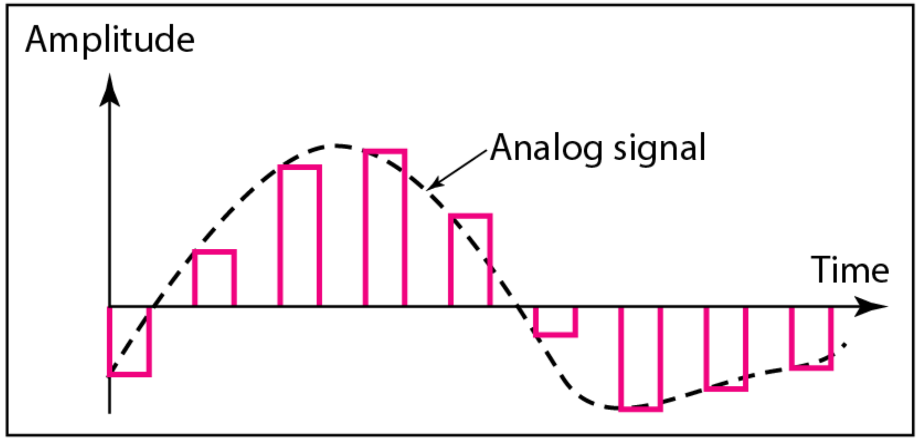
Sampling Methods



a. Ideal sampling

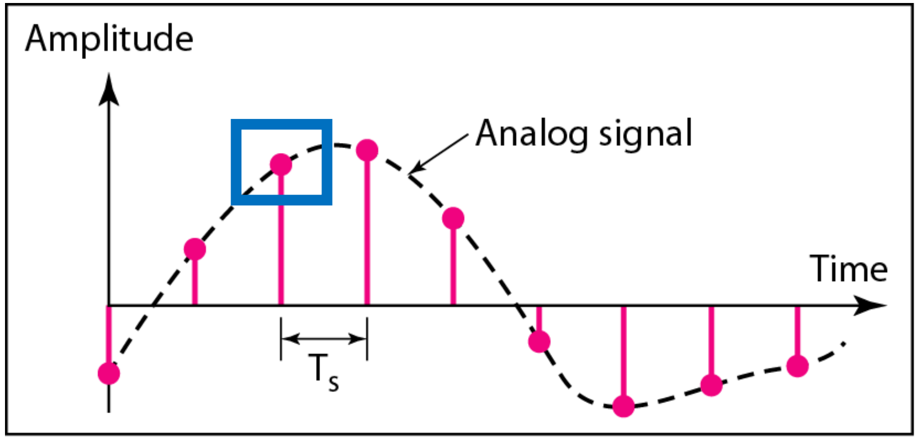


b. Natural sampling



c. Flat-top sampling **(sample and hold)**

Sampling Methods



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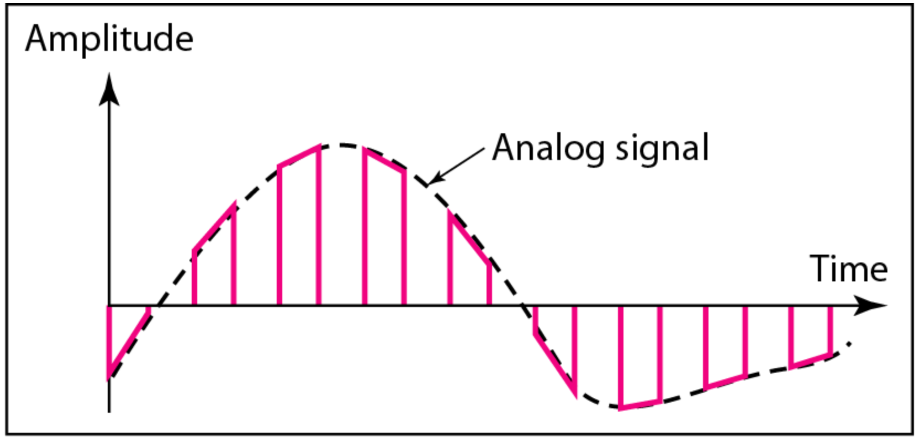

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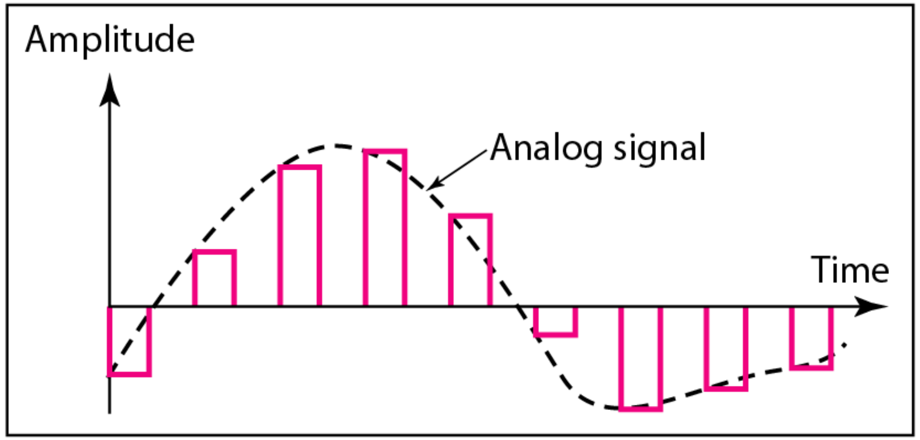

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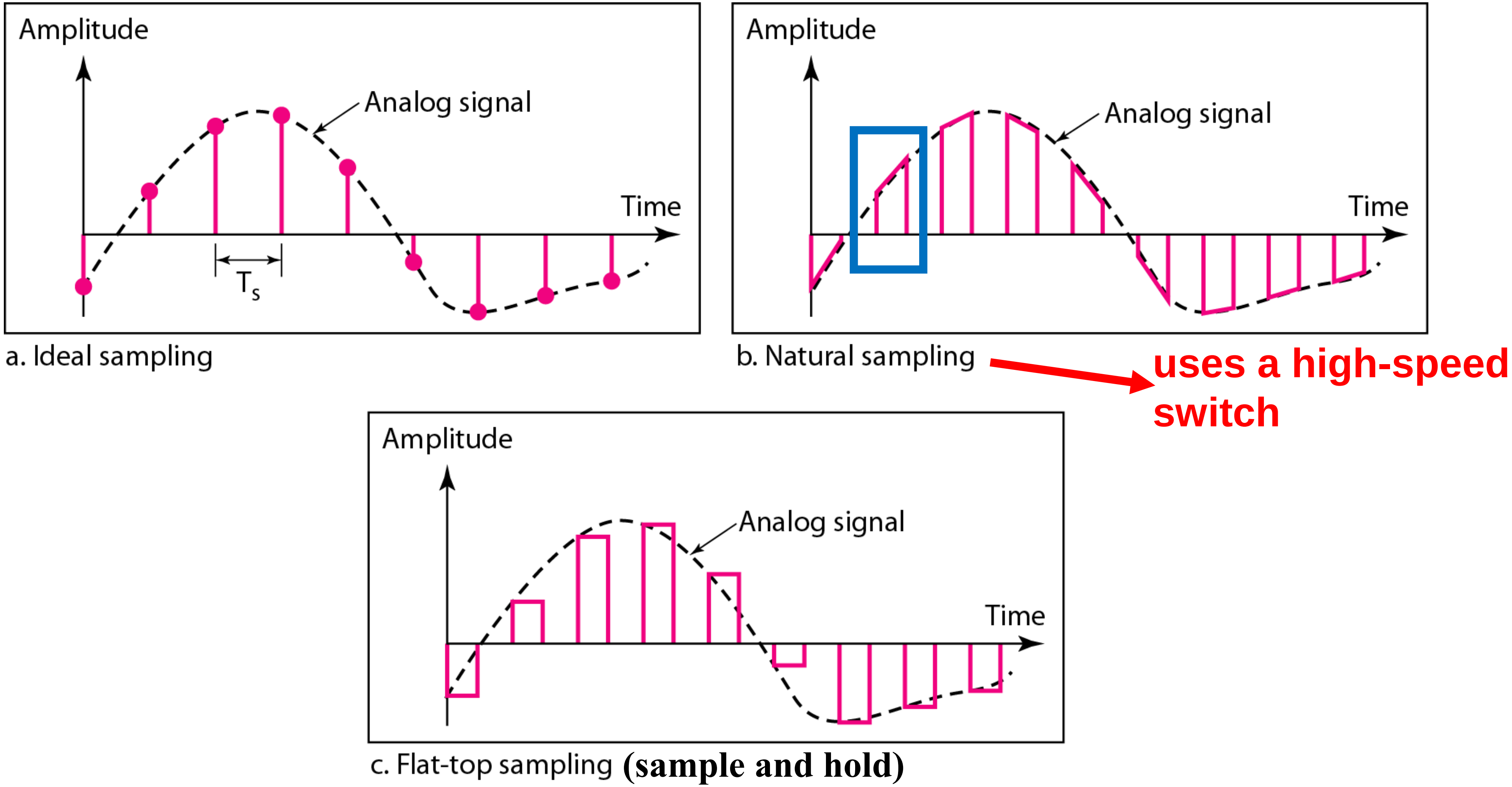

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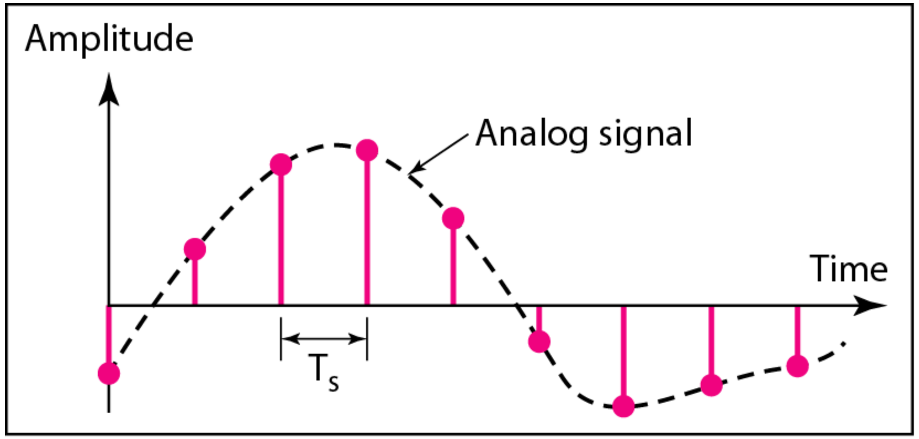

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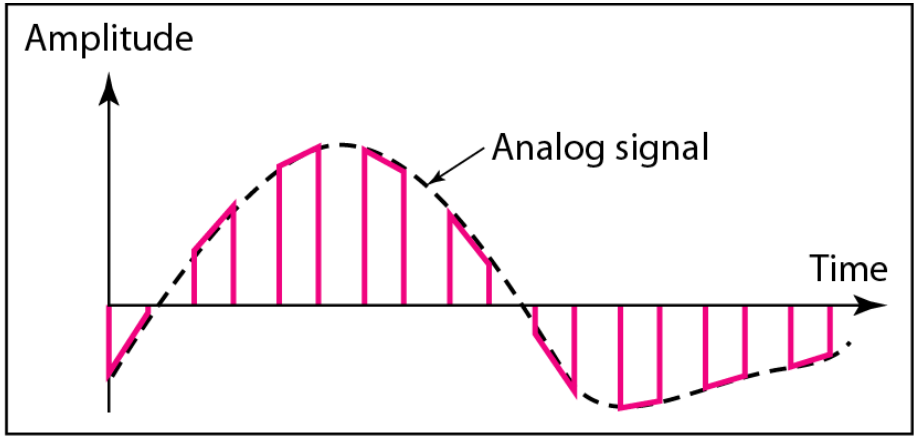

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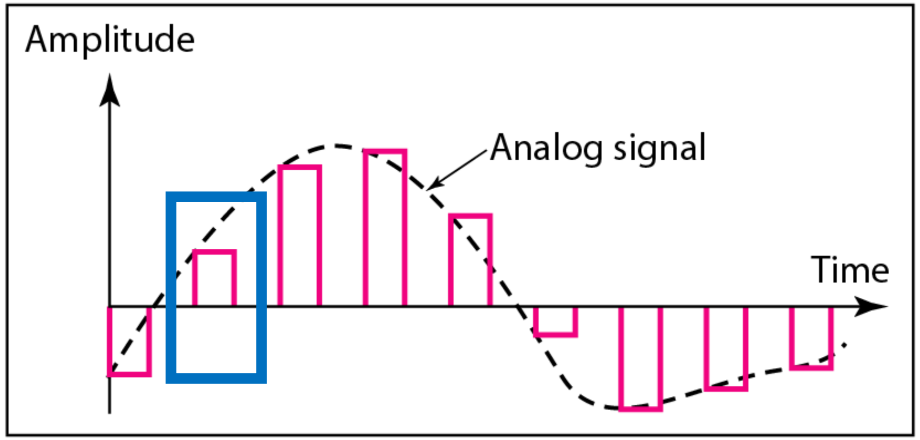
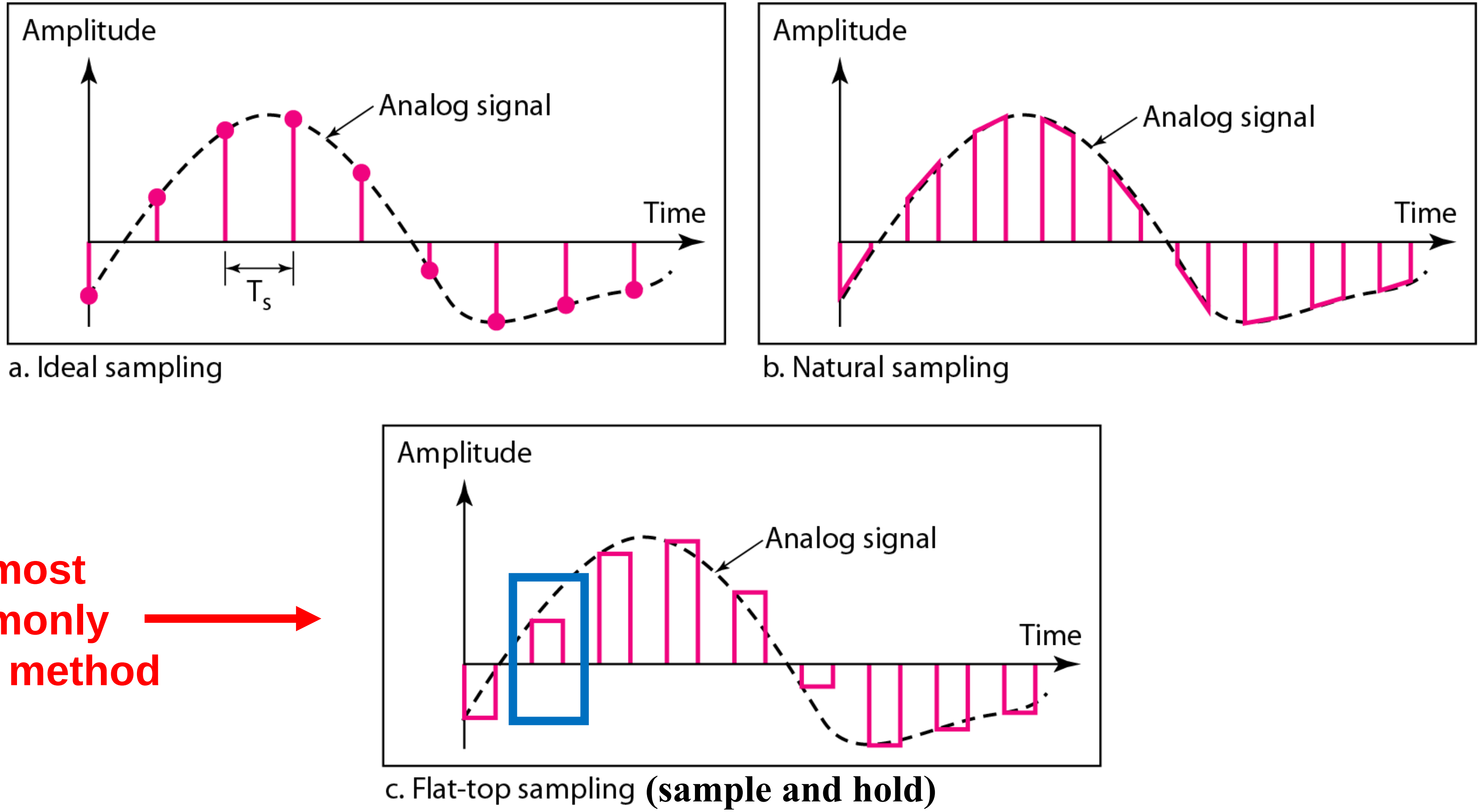


uses a circuit

c. Flat-top sampling **(sample and hold)**

Sampling Methods



(sample and hold)

The most commonly used method

Nyquist Theorem and Sampling Rate

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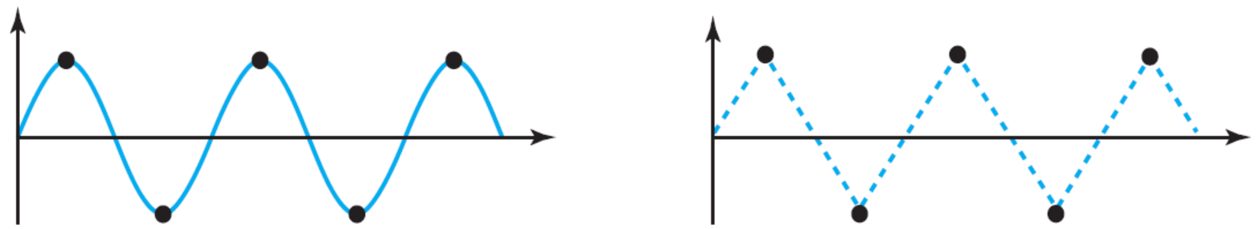
According to the Nyquist theorem, to reproduce the original analog signal, the sampling rate must be at least 2 times the highest frequency contained in the signal.

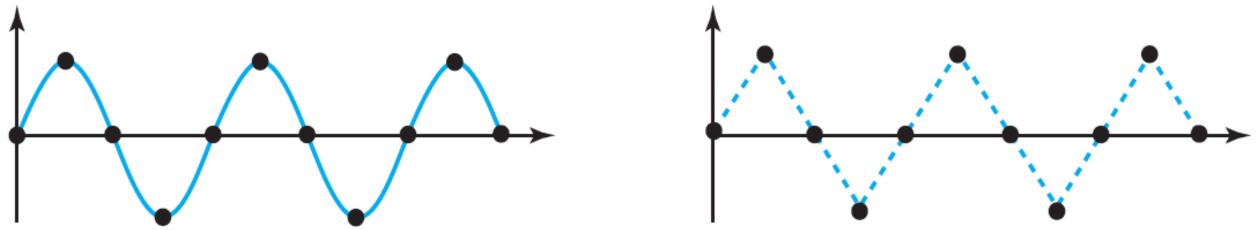
Low-pass analog signal → bandwidth = highest frequency

Bandpass analog signal → bandwidth < highest frequency

Nyquist Theorem and Sampling Rate – Example



a. Nyquist rate sampling: $f_s = 2f$



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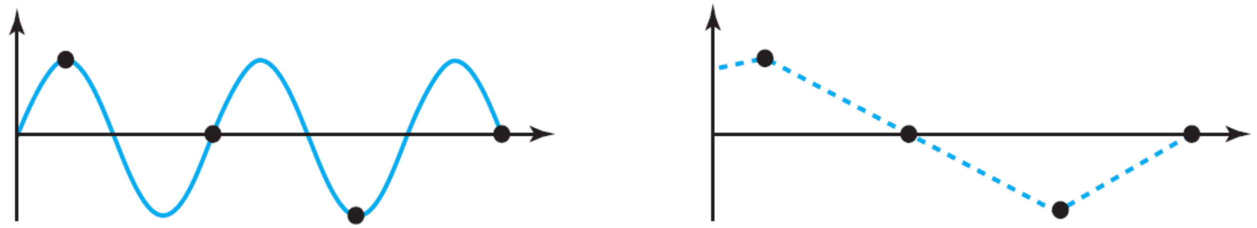

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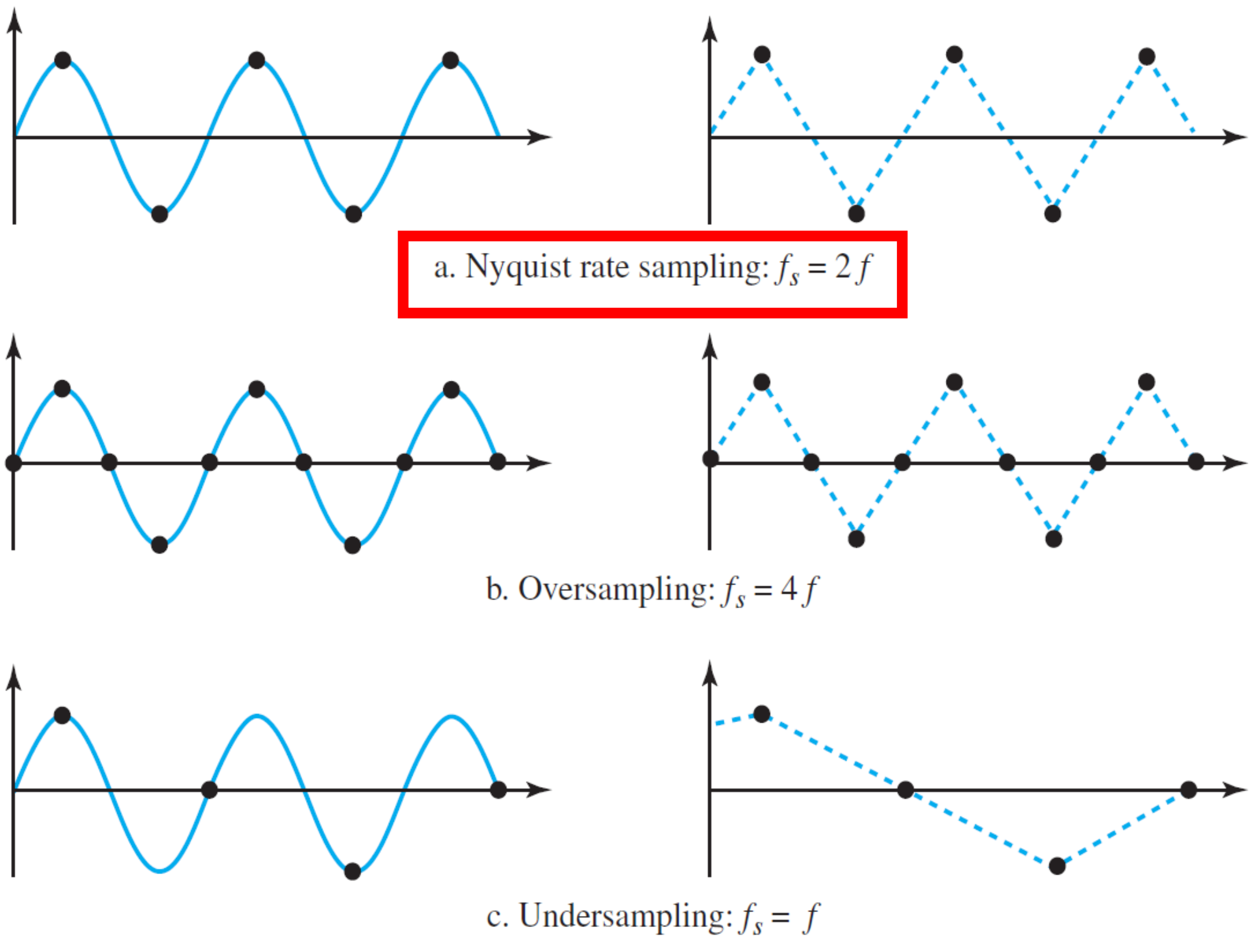
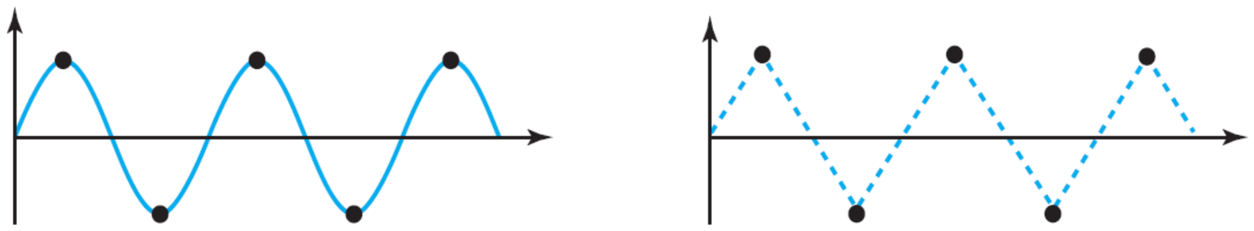
Nyquist Theorem and Sampling Rate – Example



A good approximation of the original sine wave

Nyquist Theorem and Sampling Rate – Example



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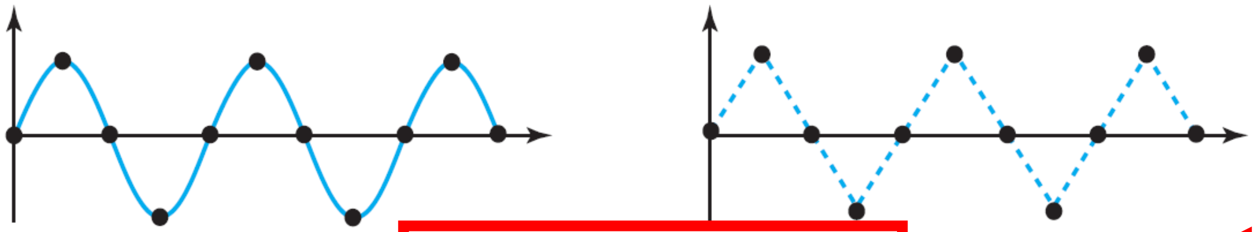

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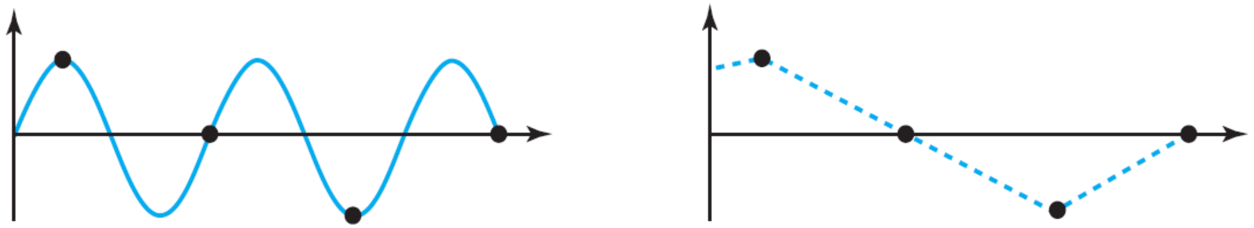
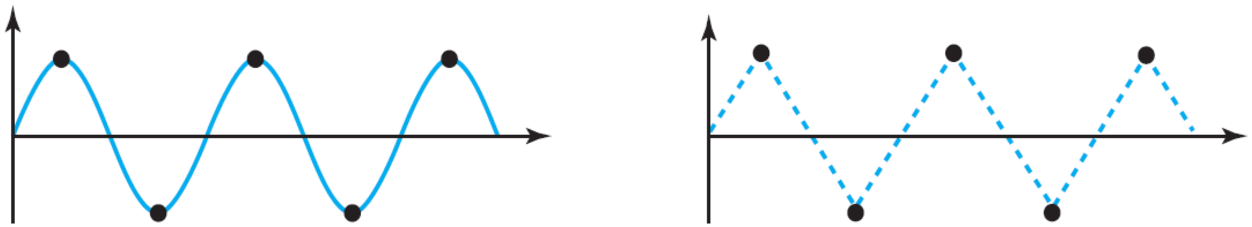


c. Undersampling: $f_s = f$

Same approximation but redundant and unnecessary

Nyquist Theorem and Sampling Rate – Example



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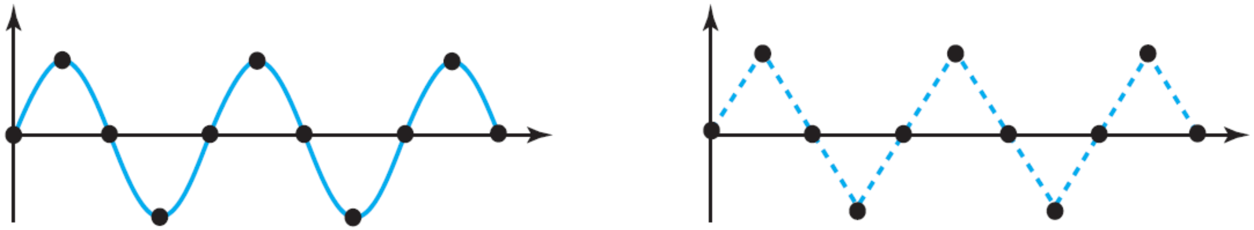

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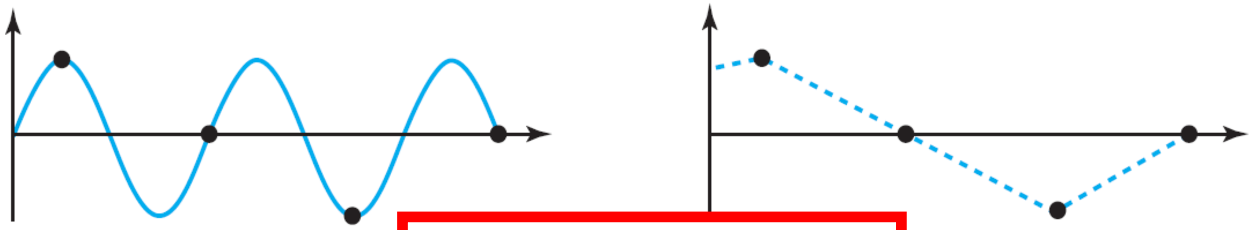

c. Undersampling: $f_s = f$

Produces a signal that does not look like the original one

Nyquist Theorem and Sampling Rate – Example

What is the minimum sampling rate for a **low-pass signal** that has a bandwidth of 100 kHz?

Nyquist Theorem and Sampling Rate – Example

What is the minimum sampling rate for a **low-pass signal** that has a bandwidth of 100 kHz?

For a low-pass signal, bandwidth = highest frequency (f_{max})

Therefore,

Minimum sampling rate = 2 x 100, 000 = 200,000 samples per second

Nyquist Theorem and Sampling Rate – Example

What is the minimum sampling rate for a **bandpass signal** that has a bandwidth of 100 kHz?

Nyquist Theorem and Sampling Rate – Example

What is the minimum sampling rate for a **bandpass signal** that has a bandwidth of 100 kHz?

We cannot find the minimum sampling rate because we do not know the maximum frequency of the signal.

Quantization

- **Quantization**
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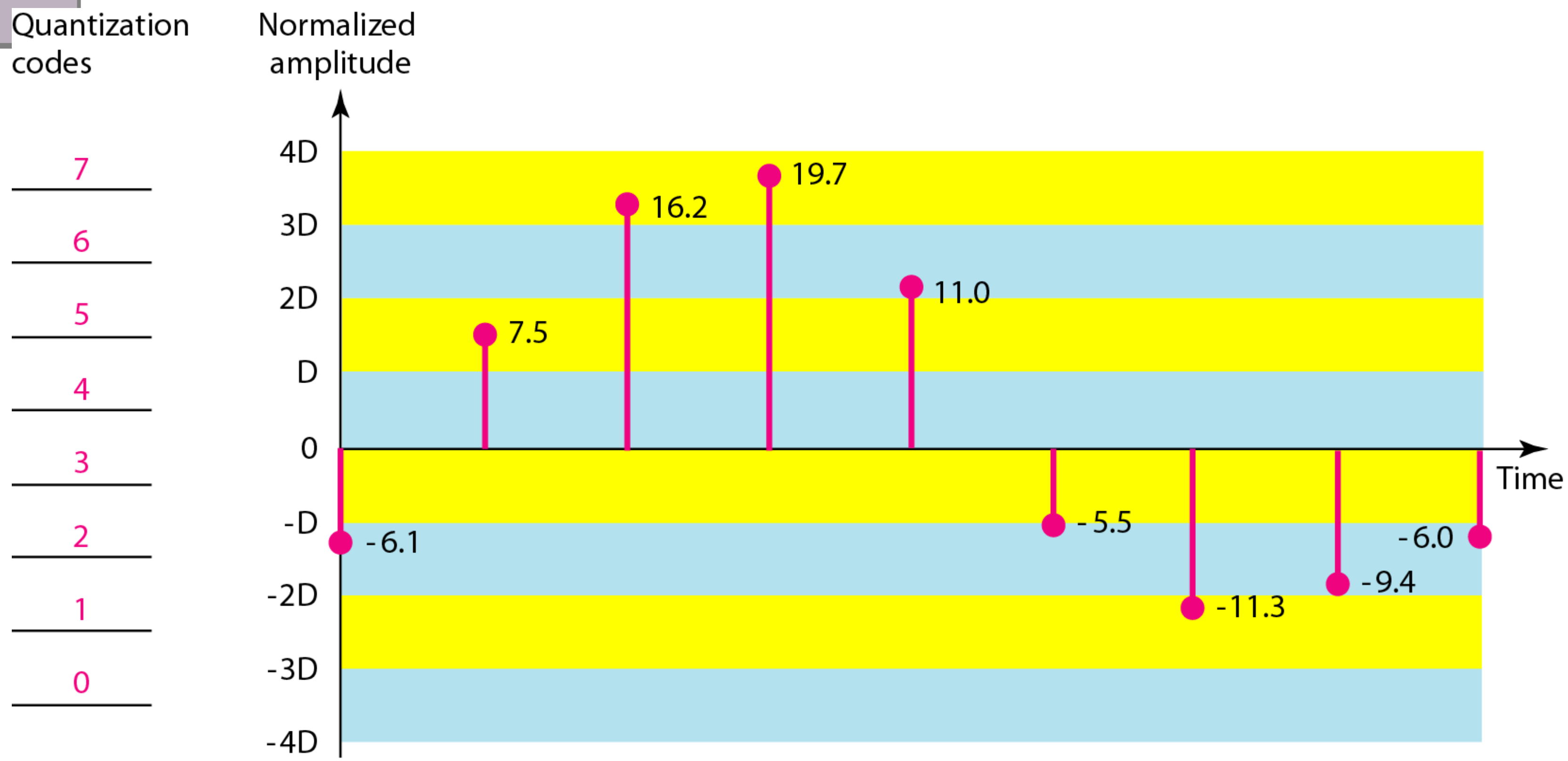
- Quantization is an approximation process.
- **Steps involved in the quantization process:**
  1) Assume that the original analog signal has instantaneous amplitudes between V_{min} and V_{max}.
  2) Divide the range into L zones (L also called the number of quantization levels), each of height Δ (delta)

$$\Delta = \frac{V_{max} - V_{min}}{L}$$

  3) Assign quantized values of 0 to $L - 1$ to the midpoint of each zone.
  4) Approximate the value of the sample amplitude to the quantized values.

Quantization - Example



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Normalized quantized values	-1.50	1.50	3.50	3.50	2.50	-1.50	-2.50	-1.50	-1.50
Normalized error	- 0.28	0	+0.26	-0.44	+0.30	-0.40	-0.24	+0.38	-0.30
Quantization code	2	5	7	7	6	2	1	2	2
Encoded words	010	101	111	111	110	010	001	010	010

Sample amplitudes are between -20 V and +20 V

L = 8, Δ = 5 V

Zones: (-20 , -15), (-15, -10), (-10, -5), (-5, 0), (0, 5), (5, 10), (10, 15), (15, 20)

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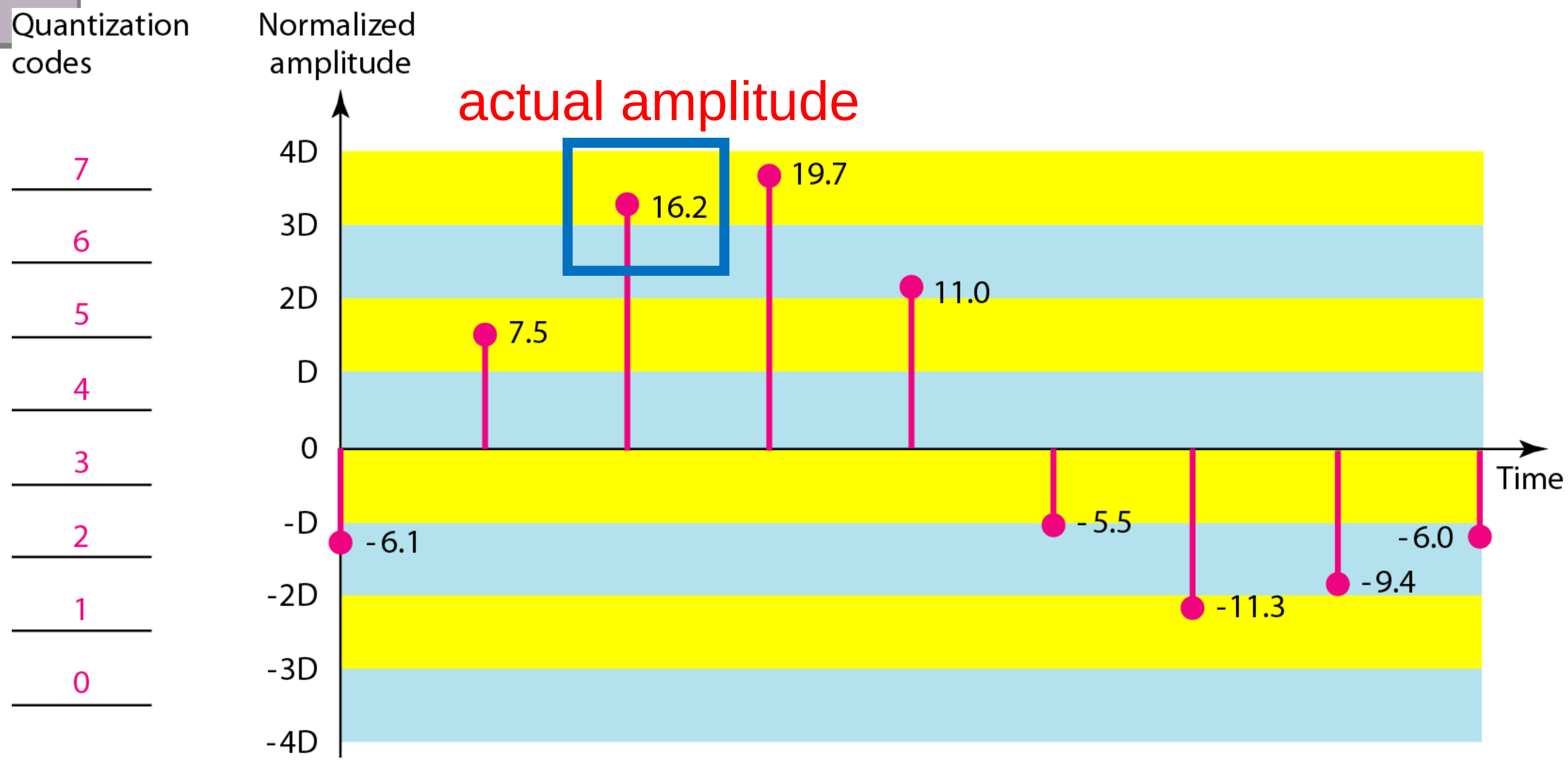

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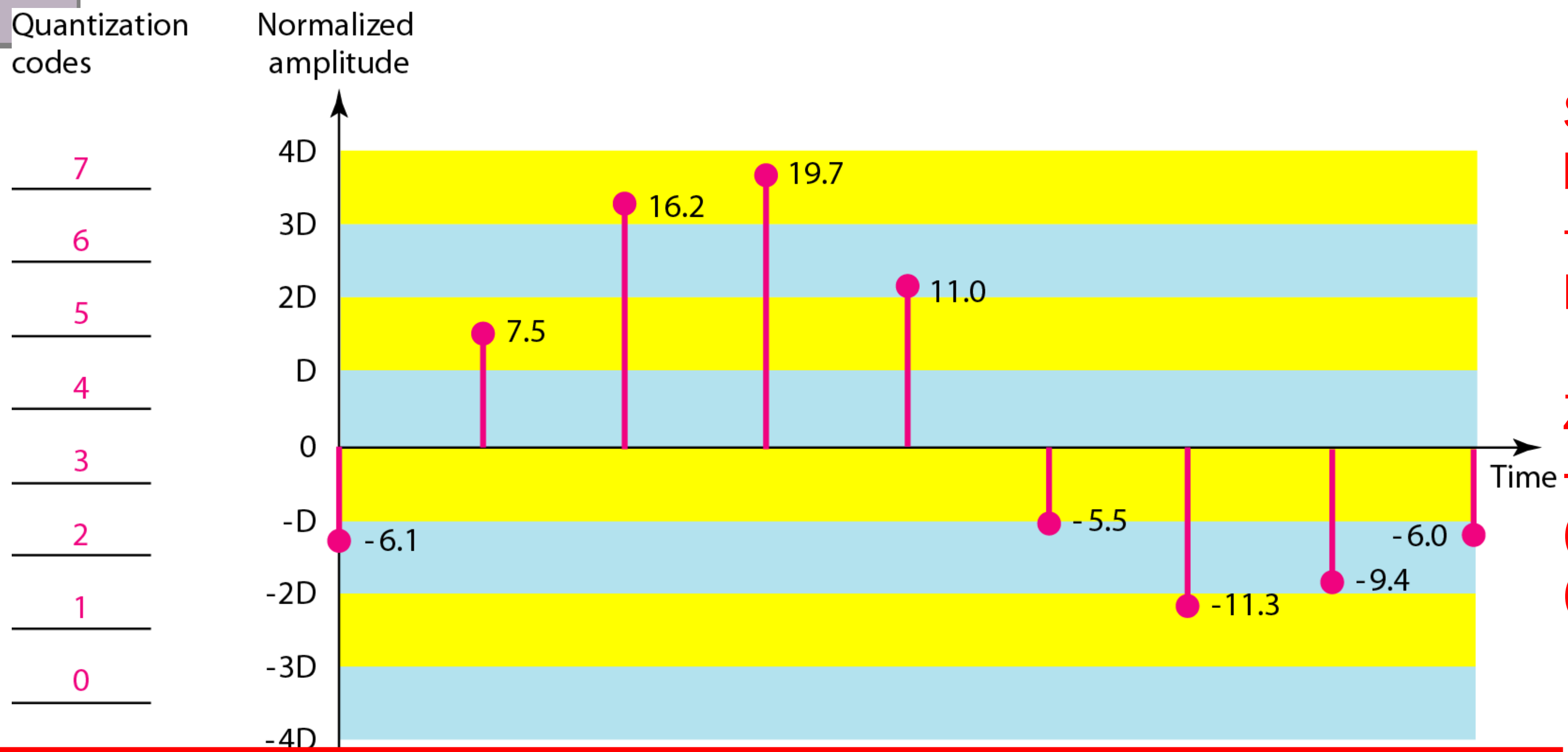

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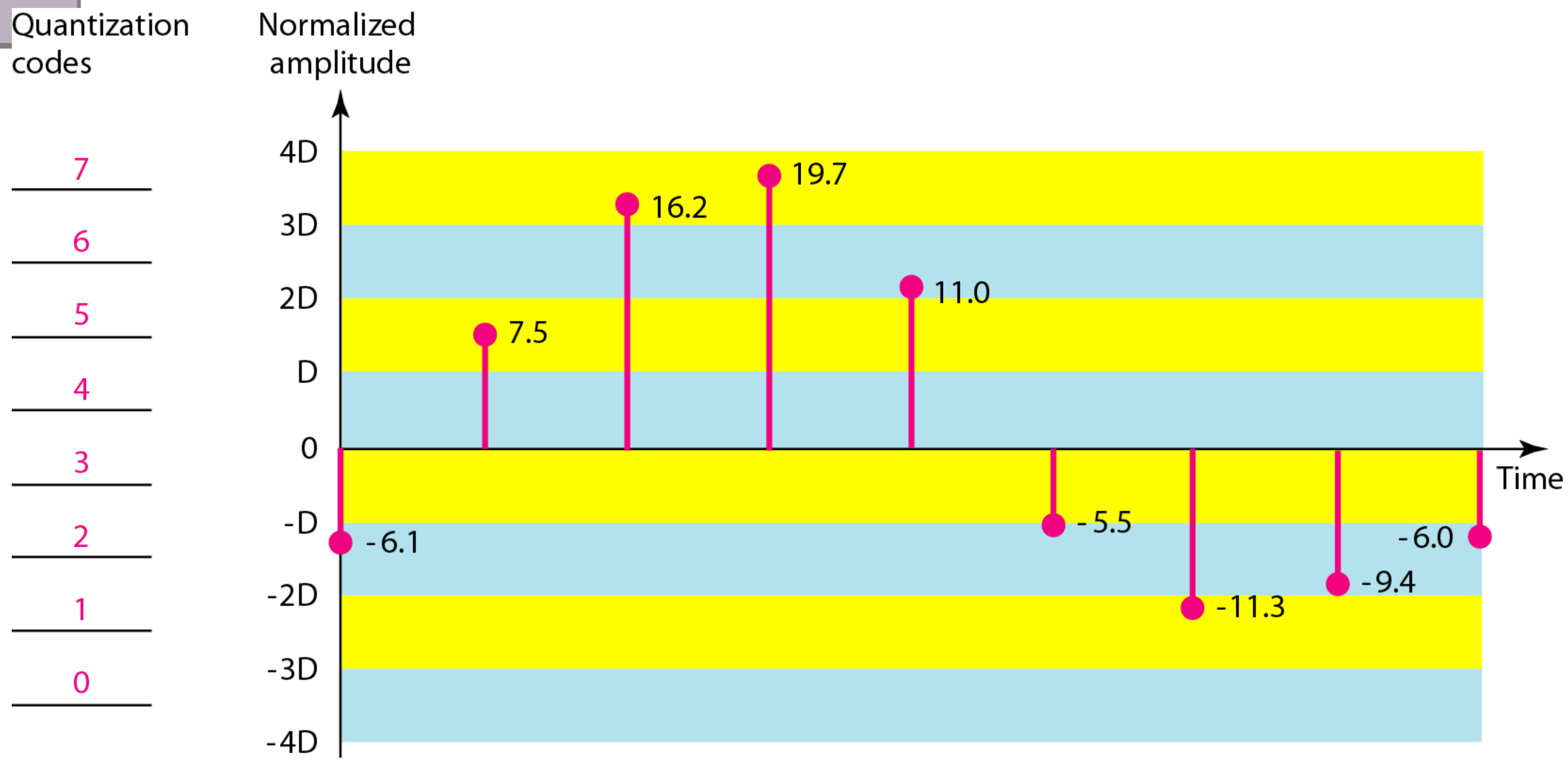
actual amplitude/Δ

e.g., -6.1 / 5 = -1.22

Quantization - Example



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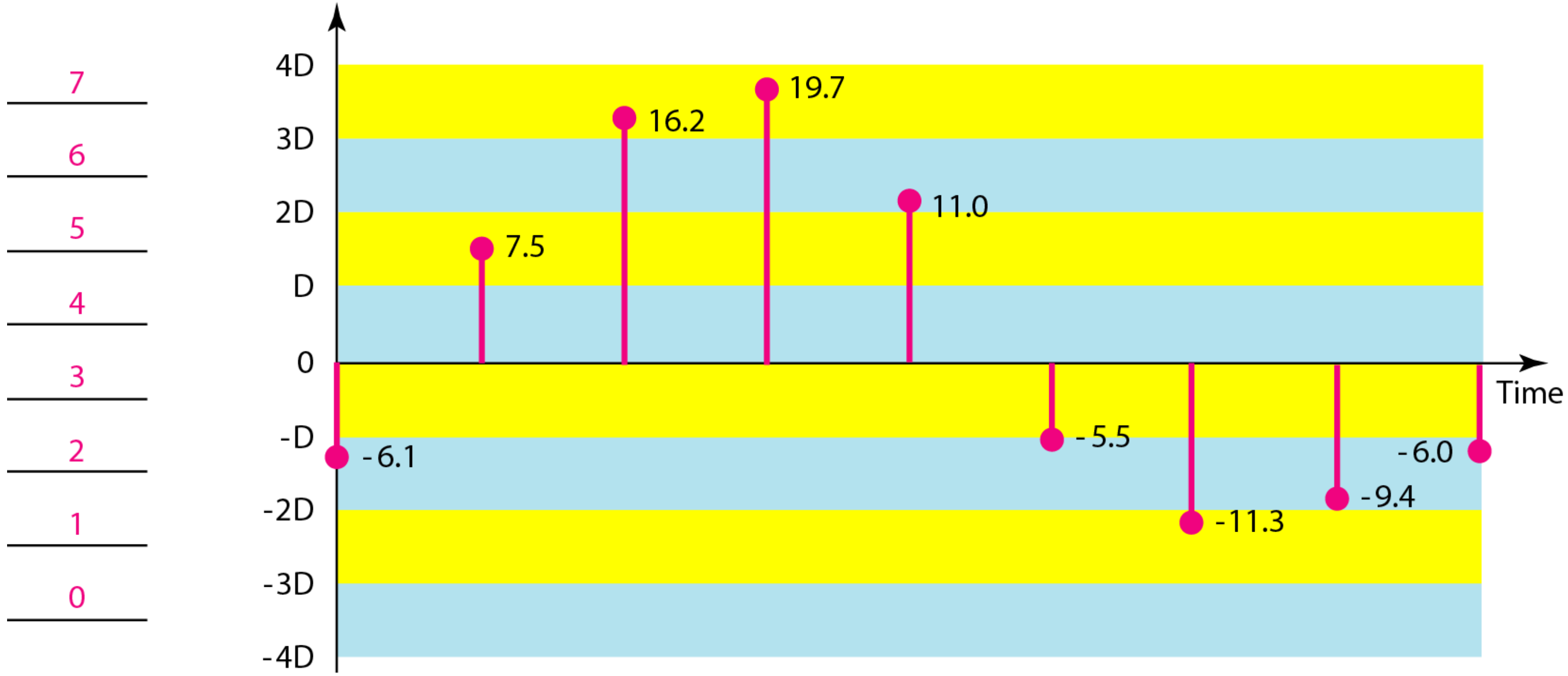
Normalize zones:
(-4, -3), (-3, -2), (-2, -1), (-1, 0), (0, 1), (1, 2), (2, 3), (3, 4)
-1.22 belongs to zone (-2, -1) → normalized quantized value = midpoint of this zone = -1.5

Quantization - Example



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Normalized error is the difference between normalized quantized value and normalized PAM value

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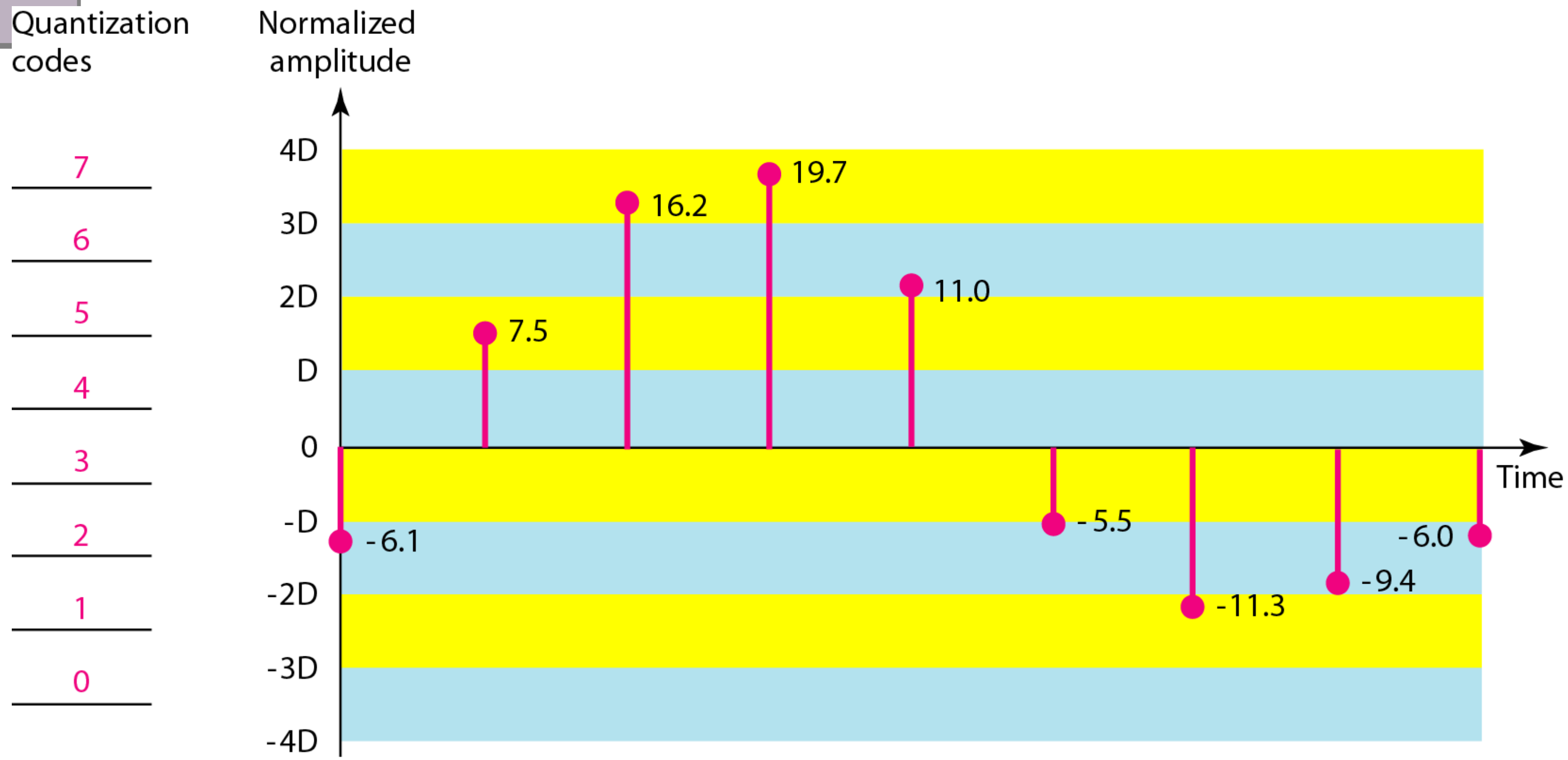

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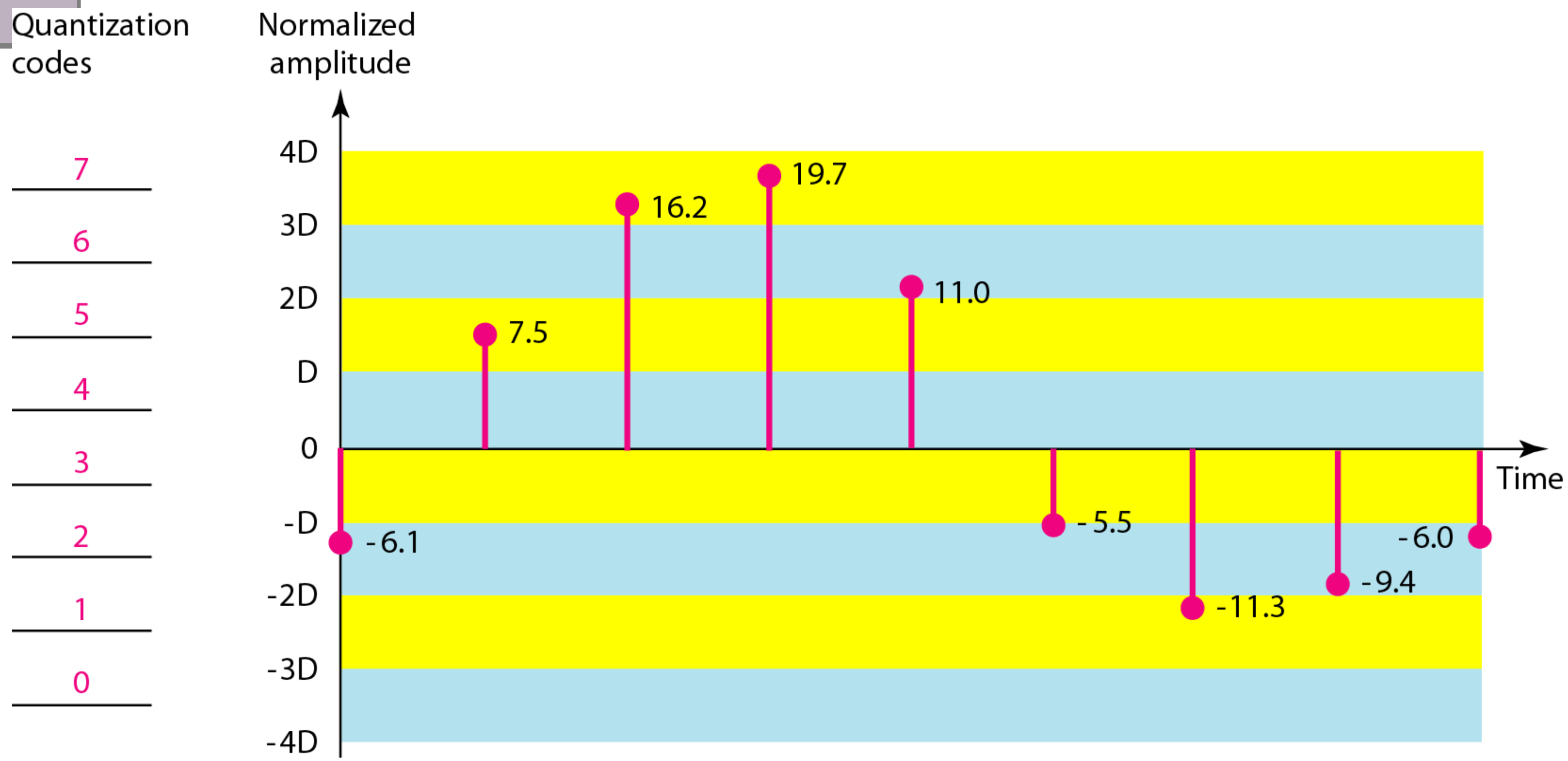
Normalized zones are mapped to 0-7: (-4, -3) → 0, (-3, -2)→ 1, (-2, -1)→ 2, (-1, 0)→3, (0, 1)→4, (1, 2)→5, (2, 3)→6, (3, 4)→7

-1.5 → midpoint of zone 2

Quantization - Example



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Encoding (Last step of PCM)

Quantization Levels

- **Choosing L (the number of quantization levels)** depends on:
  - the range of the amplitudes of the analog signal
  - the required accuracy of recovering the signal
- If the amplitude of a signal fluctuates between two values only, $L = 2$
- For a signal with many amplitude values, e.g., voice, more quantization levels are needed.
- In audio digitizing, $L = 256$
- Choosing lower values of L increases the quantization error if there is a lot of fluctuation in the signal.

Quantization Error (Noise)

- $-\Delta/2 <= $ quantization error $<= \Delta/2$
- The **quantization error changes the SNR of the signal →** the upper limit capacity is **decreased** (according to Shannon Capacity)
- The contribution of the **quantization error** to the $\mathbf{SNR_{dB}}$ of the signal depends on the number of quantization levels L, or the bits per sample n_b.

Uniform Quantization

- Issues with uniform quantization
  - only optimal for uniformly distributed signal.
  - often the distribution of the instantaneous amplitudes in the analog signal is not uniform.
  - applications such as speech and music (real audio signals) are more concentrated near zeros (lower amplitudes).
  - human ear is more sensitive to quantization errors at small values.

Encoding

- Each quantized sample can be changed to an n_b-bit code word

$n_b = \log_2 L$ (L is the number of quantization levels/zones)

Number of bits per sample = $n_b = \log_2 L$

Bit rate = sampling rate x number of bits per sample
= $f_s \times n_b$

PCM Bandwidth

- Given the bandwidth of a **low-pass analog signal**, we want to find the **new minimum bandwidth of the channel that can pass the digitized version of this signal**.

$$B_{min} = c \times N \times 1/r = c \times n_b \times f_s \times 1/r = c \times n_b \times 2 \times f_{max} \times 1/r$$

$$B_{min} = c \times n_b \times 2 \times B_{analog} \times 1/r$$

If $c = ½$ (average case) and $r = 1$ (NRZ or bipolar line coding):

$$\mathbf{B_{min} = n_b \times B_{analog}}$$

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If $c = ½$ (average case) and $r = 1$ (NRZ or bipolar line coding):

$$\mathbf{B_{min} = n_b \times B_{analog}}$$

The minimum bandwidth of the digital signal is n_b times greater than the bandwidth of the analog signal. This is the price we pay for digitization.

Agenda

- Introduction
- Analog-To-Digital Conversion
- **Summary**

Summary

- PCM is a technique for analog-to-digital conversion.
- PCM includes sampling, quantizing and encoding.
- PCM requires more bandwidth than the bandwidth of the input analog signal.

References

[1] Behrouz A.Forouzan, Data Communications and Networking, 5th Ed, 2013, McGraw-Hill companies.